REAL NUMBERS

Real numbers are the numbers that are able to accurately represent set quantities. In mathematics, they are represented by the symbol $\mathbb{R}$, or **R**.

Real numbers have the following properties:

1.) Commutative: Real numbers in a mathematical expression can be rearranged to produce the same answer (provided the operations remain the same).

$x+y=y+x$

$x(y)=y(x)$

2.) Associative: Real numbers in a mathematical expression can be grouped differently to produce the same answer.

$x+(y+z)=(x+y)+z=(x+z)+y$

$x(yz)=(xy)z=(xz)y$

3.) Identity: A real number added to zero or multiplied by one always equals itself.

$x+1=x$

$x(1)=x$

4.) Inverse: A real number added to its opposite equals zero. A real number multiplied by its inverse equals one.

$x+(-x)=0$

$x(1/x)=1$

5.) Distributive: The result of two real numbers multiplied by another real number equals the sum of the products of the numbers. The following example should make this clearer.

$x(y+z)=xy+xz$

NOTE: This property can be used in reverse.

$xy+xz=x(y+z)$

Make sure to master the use of this property, as it enables you to manipulate equations very easily, which is invaluable in algebra.

The category of real numbers is actually a very broad group. On that account, it has been divided into many other groups. We will start with the least inclusive, Natural, or Counting numbers.

Natural/Counting Numbers

These are really easy. The set of Natural numbers begins at one, and includes every whole number greater than one, going on to infinity. It is represented by the symbol **N**.

Ex. Real numbers: 3, 72, 188 million

Ex. **NOT** real numbers: 0, -4, .26

Whole Numbers

This is the next group. It is the set of Natural numbers and zero. It is represented by the symbol **W**. It also goes on to infinity.

Ex. Whole numbers: 0, 65, 144 Ex. **Not** real numbers: -73, .5

Integers

Integers are the next step. Represented by the letter **I**, this set includes every whole number, plus the opposite of every natural number. Said differently, it consists of every whole number, positive and negative, from negative infinity to infinity.

Ex. Integers: -7, 44, -667, 0 Ex. **Not** integers: .5, pi

Rational Numbers

The next group is the group known as the rational numbers. It is represented by the letter **Q**. Any rational number can be accurately defined as a ratio of two numbers (i.e. 3 equals 3/1, and .5 equals ½). Rational numbers also always have terminating decimals. However, be careful not to assume a number is rational because it has a terminating approximation. A good example is pi. Pi is an irrational number that cannot be defined by a ratio, and has a non terminating decimal. However, the approximation 3.14 is often used for pi. 3.14 is a rational number, but pi is not.

Ex. Rational numbers: .4456, 92, -4.83 Ex. **Not** integers: pi, √2

Irrational Numbers

Irrational numbers are the rest of the real numbers. They are defined as any number that is not accurately represented by a ratio. They have non-terminating decimals. Thus, the group of irrational numbers does not include rational numbers, and therefore also does not include, natural numbers, whole numbers, or integers.

This diagram should help explain what was just covered.

THE NUMBER LINE

Dealing with the number line is easy. You simply have to have a good grasp of a few simple concepts, and some notations.

A number line is an abstract idea, a line from negative infinity to infinity that includes every real number, which are all represented by a single point on the line. This is a very useful tool for representing sets.

One way that sets are represented are by giving the name of the set, followed by an equals sign, then either a parentheses or bracket, then the set, then another parentheses or bracket to close the statement. Brackets are inclusive, meaning that the set includes the number it is next to. Parentheses are exclusive, meaning the set includes every number up to the number it is next to. Three dots at the beginning of a list means "from negative infinity," and three dots at the end of a list means "to infinity."

N, the set of whole numbers is expressed like this:

N=[1,2,3,4…]

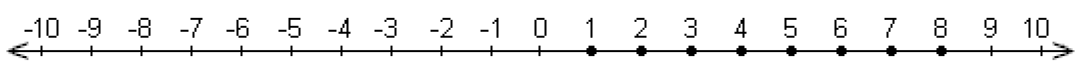
In this instance, the first number of the set, 1, is included, so it is surrounded by a bracket. Then the next few numbers are given to establish the pattern that defines the set (in this case, every whole number greater than zero). Finally, the three dots and the bracket at the end signify that the set includes every number following that pattern to infinity.

Suppose there was a set **P** that included every whole number up to 8. That set would be defined like this:

P=[0,1,2,3,4,5,6,7,8,]

Since all members of **P** are whole numbers, but **P** does not include all whole numbers, **P** is known as a subset of the set of all whole numbers **W**.

On a number line, **P** would look like this:

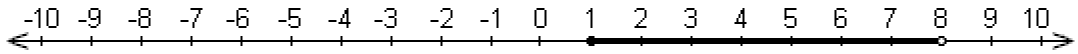


The solid black dot means that that point is included in the set.

Sets can also include every point on a number line between two given points. To define a set like this, one would use the following notation. For a hypothetical set **X** that includes all the points from 1 up to, but not including 8, one would say:

X=[1,8)

This says that the first number of the set is 1, and that it includes all real numbers (and therefore all points on the number line) up to 8. However, 8 is not part of this set, as we know because of the parentheses on the end. On a number line, **X** would look like this:



The solid dot at 1 signifies that one is the first value in the set. The solid line means that every value between one and 8 is included in the set. The open dot at 8 means that 8 is excluded from the set.

ORDER OF OPERATIONS

The order of operations is a set of rules that regulates the way mathematical statements are solved, so that each statement has one unique solution. For example, $4+5\times2$ could equal 18 or 14, depending on how it was simplified.

If one adds first,
$4+5\times2$
$9\times2=18$

If one multiplies first,
$4+5\times2$
$4+10=14$

The second example is correct, because it correctly follows the order of operations.
To correctly solve a problem, one must simplify it in this order:

1.) Parentheses- simplify what is inside them first, no matter what it is
2.) Exponents- simplify exponents to eliminate them
3.) Multiplication/Division- like in the example, do these operations before adding
4.) Addition/Subtraction- finish up by adding and subtracting

A pneumonic device to remember this order is "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally."

Here is an example problem

$(4+(3+7)^2)2$
$(4+(10)^2)2$ Simplify inside the innermost parentheses first, even though it is only addition
$(4+100)2$ Simplify the exponent
$(104)2$ Now you can simplify the next innermost set of parentheses
208 Since there are no more functions inside parentheses or any more exponents, multiply. Since there is no adding to be done, this is your solution

Note that we did some adding before we multiplied or simplified exponents. That is because we always have to simplify parentheses first, and the only operation in the parentheses was addition. Had there been any multiplication, division, or exponents in the parentheses as well, we would have done those before the addition. You see that scenario in next example.

$(18-(3+4^3\times2))4$ We see that there are 2 sets of parentheses, so we work on the inside one
$(18-(3+64\times2))4$ Simplify the exponent, since that is the priority in this set of parentheses
$(18-(3+128))4$ Now we can multiply
$(18-(131))4$ Since addition is the only operation left inside the parentheses, we do it
$(-113)4$ Since we simplified the first set of parentheses, we can work on the next

-452 Simplify what is left for the final answer

It helps to think of parentheses as isolated statements that require the use of the order of exponents. Once they are simplified, you move on to the next set of parentheses, able to use the set you just simplified as a single number.

EXPONENTS

Exponents are a way of saying "a number times itself." For example,

$2^3=2\times2\times2$

IT DOES NOT EQUAL 2×3!

Exponents can also work in the opposite direction. For example, finding a square root of a number x is the same as asking "What number times itself gives x?" Similarly, finding a third root of a number x is the same as asking "What number times itself times itself gives me x?" and so on.

Notation:

In x^y, x is the base, y is the exponent. This expression can be said "x to the y," or "x raised to the power of y."

Some basic rules of exponents:

1.) When multiplying powers with the same base, add the exponents

$x^y\times x^z\times=x^{y+z}$

2.) When dividing powers with the same base, subtract the exponents

$x^y\div x^z\times=x^{y-z}$

3.) When raising a power to a power, multiply the exponents

$(x^y)^z=x^{yz}$

4.) When raising a product to a power, distribute the exponent

$(xy)^z=x^zy^z$

5.) When raising a quotient to a power, distribute the exponent

$(x/y)^z=(x^z/y^z)$

6.) Any nonzero number raised to the power of zero is 1.

$572^0=1$

That is a confusing idea. So this is how it works:

We know that any number divided by itself equals 1. We also know that $x^a\div x^a=x^{a-a}$ which equals x^0. Thus, we know that any number to the power of zero equals one.

7.) Negative exponents cause a number to be "flipped" or to become its reciprocal.

$(x/y)^{-1}=y/x$

$3^{-1}=1/3$

$(x/y)^{-3}=(y^3/x^3)$

8.) When the exponent is a fraction, raise the base to the power of the numerator and take the root to the power of the denominator

$x^{3/2}=\sqrt{x^3}$